S–253(A to C)

B. A./B. Sc. (Sixth Semester)

EXAMINATION, 2019

MATHEMATICS

Time : Two Hours] [*Maximum Marks : 70*

S–253(A)

(Numerical Methods)

[SOS/Maths./DSE—002(A)]

नोट : (i) खण्ड 'अ' से किन्हीं पाँच प्रश्नों के और खण्ड 'ब' से किन्हीं तीन प्रश्नों के उत्तर दीजिए।

Attempt any *five* questions from Section A and any *three* questions from Section B.

(ii) खण्ड 'अ' के प्रत्येक प्रश्न का उत्तर 50 शब्दों तक सीमित रखें।

Answer each question of Section A within 50 words.

(iii) अपने सभी प्रश्नों के उत्तर आपको दी गयी उत्तर पुस्तिका में ही दीजिये। अतिरिक्त उत्तर पुस्तिका नहीं दी जायेगी।

Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

खण्ड—अ

(Section—A)

नोट : किन्हीं पाँच प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न 5 अंक का है।

Attempt any *five* questions. Each question carries 5 marks.

1. दिया हुआ है :

$\log 100 = 2$, $\log 101 = 2.0043$, $\log 103 = 2.0128$ एवं $\log 104 = 2.0170$।

$\log 102$ का मान ज्ञात कीजिए।

Given that :

$\log 100 = 2$, $\log 101 = 2.0043$, $\log 103 = 2.0128$ and $\log 104 = 2.0170$.

Find $\log 102$.

2. दर्शाइए कि :

$$\Delta \log f(x) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$$

Show that :

$$\Delta \log f(x) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$$

3. दिया हुआ है कि $f(0) = 8, f(1) = 68, f(5) = 123$ । एक डिवाइडेड डिफरेंस तालिका का निर्माण कर इसकी सहायता से $f(2)$ का मान ज्ञात कीजिए ।

Given that $f(0) = 8, f(1) = 68, f(5) = 123$. Construct a divided difference table and by using it find the value of $f(2)$.

4. निम्नलिखित तालिका से $f'(4)$ का मान ज्ञात कीजिए :

x	$f(x)$
1	0
2	1
4	5
5	27

From the following table determine the value of $f'(4)$:

x	$f(x)$
1	0
2	1
4	5
5	27

5. निम्नलिखित तालिका दी गई है :

x	$f(x)$
0	3
1	6
2	11
3	18
4	27

फलन का रूप क्या है ?

The following table is given :

x	$f(x)$
0	3
1	6
2	11
3	18
4	27

What is the form of function ?

6. निम्नलिखित तालिका से $f(3.5)$ का मान स्टर्लिंग सूत्र से ज्ञात कीजिए :

x	$f(x)$
1	1
2	4
3	9
4	16
5	25
6	36
7	49

From the following table, find the value of $f(3.5)$ by Stirling formula :

x	$f(x)$
1	1
2	4
3	9
4	16
5	25
6	36
7	49

7. सिद्ध कीजिए कि :

$$\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$$

Prove that :

$$\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$$

खण्ड—ब

(Section—B)

नोट : किन्हीं तीन प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न 15 अंक का है।

Attempt any *three* questions. Each question carries 15 marks.

8. (अ) एक परीक्षा में छात्रों, जिन्होंने निश्चित सीमाओं में अंक प्राप्त किये थे, की संख्या अग्रलिखित थी :

9.

अंक	छात्रों की संख्या
0—19	41
20—39	62
40—59	65
60—79	50
80—99	17

उन छात्रों की संख्या का आकलन कीजिए जिन्होंने 70 से कम अंक प्राप्त किये।

In an examination, the number of students, who obtained marks between certain limits, were as follows :

Marks	No. of Students
0—19	41
20—39	62
40—59	65
60—79	50
80—99	17

Estimate the number of students who obtained less than 70 marks.

(ब) न्यूनतम सम्भव डिग्री का वह बहुपद प्राप्त कीजिए जिसका $x = 3, 2, 1, -1$ पर मान क्रमशः 3, 12, 15, –21 है।

Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, –21, when $x = 3, 2, 1, -1$ respectively.

9. (अ) समीकरण $x^5 + 5x + 1 = 0$ के मूल दशमलव के तीन स्थानों तक न्यूटन-रैफ्सन विधि द्वारा ज्ञात कीजिए।

Find the roots of the equation $x^5 + 5x + 1 = 0$ correct to the three decimal places by Newton-Raphson method.

(ब) सिम्पसन $\frac{1}{3}$ वें नियम द्वारा $\int_0^6 \frac{dx}{1+x^3}$ का मान ज्ञात कीजिए।

Evaluate $\int_0^6 \frac{dx}{1+x^3}$ by using Simpson's $\frac{1}{3}$rd rule.

10. (अ) सिद्ध कीजिए कि :

$$\underset{yz}{\overset{2}{\Delta}} x^3 = x + y + z$$

Prove that :

$$\underset{yz}{\overset{2}{\Delta}} x^3 = x + y + z$$

(ब) ट्रैपेजोइडल नियम की सहायता से निम्नलिखित का मान दशमलव के दो सही स्थानों तक ज्ञात कीजिए :

$$\int_0^1 \frac{dx}{1+x^2}$$

By means of Trapezoidal rule, compute the following integral with two correct decimal places :

$$\int_0^1 \frac{dx}{1+x^2}$$

11. (अ) न्यूटन के अन्तर्वेशन के अग्रसर अन्तर सूत्र को स्थापित कीजिए।

Establish Newton's forward difference formula for interpolation.

(ब) गाउस-सीडल इटरेशन विधि से निम्नलिखित समीकरण हल कीजिए :

$$8x - 3y + 2z = 20$$

$$6x + 3y + 12z = 35$$

और $$4x + 11y - z = 33$$

Solve the Gauss-Seidel iteration method for system of equations :

$$8x - 3y + 2z = 20$$

$$6x + 3y + 12z = 35$$

and $$4x + 11y - z = 33$$

12. संख्यात्मक अवकलन को परिभाषित कीजिए। न्यूटन के अग्रज अन्तर्वेशन सूत्र का प्रयोग करते हुए इसके विभिन्न घातों के अवकलन सूत्रों को ज्ञात कीजिए।

Define numerical differentiation. Find the various order derivatives formulae using Newton's forward formula for interpolation.

13. (अ) पुनरावृत्त विधि का प्रयोग करते हुए समीकरण :

$$f(x) = x^3 - 2x^2 - 4 = 0$$

का वास्तविक हल दशमलव के तीन स्थानों तक ज्ञात कीजिए।

Find the real root of the equation :

$$f(x) = x^3 - 2x^2 - 4 = 0$$

correct to three places of decimals by iteration method.

(ब) लैग्रांज सूत्र द्वारा सिद्ध कीजिए कि :

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

By means of Lagrange's formula, prove that :

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

S–253(B)

(Complex Analysis)

[SOS/Maths./DSE—002(B)]

Note : (i) Attempt any *five* questions from Section A and any *three* questions from Section B.

(ii) Answer each question of Section A within 50 words.

(iii) Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

Section—A

Note : Attempt any *five* questions. Each question carries 5 marks.

1. If $f(z) = \sqrt{|xy|}$, then show that $f(z)$ is not analytic at the origin Cauchy-Riemann equations are satisfied at that point.

2. Prove that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. Also find its harmonic conjugate.

3. If $f(z) = \frac{iz}{2}$ in the open disk $|z| < 1$, then by Σ-δ definition of limit, show that :

$$\lim_{z \to 1} f(z) = \frac{i}{2}$$

4. State and prove Cauchy's theorem (by using Green's theorem).

5. Evaluate the following integral by using the Cauchy's integral formula :

$$\int_C \frac{(\sin \pi z^2 + \cos \pi z^2)\, dz}{(z-1)(z-2)}$$

where C is the circle $|z| = 3$.

6. If C is closed contour containing the origin inside it, then prove that :

$$\frac{a^n}{\lfloor n} = \frac{1}{2\pi i}\int_C \frac{e^{az}}{z^{n+1}}\, dz$$

7. Prove that :

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z\, \partial \bar{z}}$$

where $z = x + iy$.

Section—B

Note : Attempt any *three* questions. Each question carries 15 marks.

8. (a) Show that an analytic function with non-zero constant modulus is constant.

(b) If the function w is defined by the following :

$$z = e^{-v}(\cos u + i \sin u),\ w = u + iv.$$

then find the values of z for which the function w is not analytic.

9. Find the value of the integral :

$$\int_0^{1+i} (x - y + ix^2)\, dz$$

(a) Along the straight line from $z = 0$ to $z = 1 + i$.

(b) Along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

10. State and prove Cauchy's integral formula.

11. Verify Cauchy's theorem for the function :

$$f(z) = 4 \sin 2z$$

if C is the square with vertices at $z = 1 \pm i,\ -1 \pm i$.

12. State and prove Liouville's theorem.

13. If $f(z) = u + iv$, is a regular function of z in any domain, then prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|u|^p = p(p-1).|u|^{p-2}.|f'(z)|^2$$

S–253(C)

(Linear Programming)

[SOS/Maths./DSE—002(C)]

Note : (i) Attempt any *five* questions from Section A and any *three* questions from Section B.

(ii) Answer each question of Section A within 50 words.

(iii) Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

Section—A

Note : Attempt any *five* questions. Each question carries 5 marks.

1. If $f(x)$ and $g(x)$ be convex functions on a convex set S. Prove that their sum is also a convex function over S.

2. Define theory of Simplex method in linear programming problem.

3. Show that $x_1 = 4$, $x_2 = 0$, $x_3 = 0$, $x_4 = 4$ is an optimal basis feasible solution to the LPP :

   Max. :

$$z = x_1 + 2x_2 + 0.x_3 + 0.x_4$$

Subject to :

$$x_1 + 2x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

4. Explain the concept of Simplex algorithm.
5. What is the concept of duality ? Discuss relationship between primal and its dual.
6. Obtain the dual of the following problem :

Minimize :

$$z = x_1 + x_2 + x_3$$

Subject to :

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$$x_1, x_2 \geq 0$$

and x^3 unrestricted.

7. Define the following :

(a) Convex set

(b) Big-M method

Section—B

Note : Attempt any *three* questions. Each question carries 15 marks.

8. Prove that :

(a) The union of two convex sets is also a convex set.

(b) If $C = \{(x_1, x_2) : 2x_1 + 3x_2 = 7\} \subset R^2$ is a convex set.

9. Find the dual problem of the following primal problem :

Minimize :

$$z = x_1 - 3x_2 - 2x_3$$

Subject to the constraints :

$$3x_1 - x_2 + 2x_3 \le 7$$

$$2x_1 - 4x_2 \ge 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

$x_1, x_2 \ge 0$ and x_3 is unrestricted.

10. Show that the set of all optimal solutions to the convex problem is convex.

11. Prove that the dual of the dual is the primal in linear programming problem.

12. Use the two-phase method to :

Maximize :

$$z = 5x_1 + 3x_2$$

Subject to the constraints :

$$2x_1 + x_2 \leq 1$$

$$x_1 + 4x_2 \geq 6$$

$$x_1, x_2 \geq 0.$$

13. Solve the following problem using the Big-M method :

Maximize :

$$z = 6x_1 - 3x_2 + 2x_3$$

Subject to :

$$2x_1 + x_2 + x_3 \leq 16$$

$$3x_1 + 2x_2 + x_3 \leq 18$$

$$x_2 - 2x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0.$$